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- Non-Linear Algebraic Equations:

$$\mathbf{F}(\mathbf{X}) = \mathbf{0}; \quad \text{unknown vector: } \mathbf{X}$$

- Newton's method

$$\mathbf{X}^{k+1} = \mathbf{X}^k - \mathbf{B}(\mathbf{X}^k)^{-1} \mathbf{F}(\mathbf{X}^k)$$

$$B_{ij} = \frac{\partial F_i}{\partial X_j}; \quad \text{Tangent Stiffness.}$$

- Requires an inversion of B in each iteration
- Sensitive to the initial guess of X
- When B is singular, Arc-Length methods are needed

- Liu and Atluri (2008)

$$\dot{\mathbf{X}}(t) = \frac{-\nu}{q(t)} \mathbf{F}(\mathbf{X}) \equiv \mathbf{f}(\mathbf{X}, t) \quad -\nu \neq 0$$

- Special case: $q(t) = 1 + t$
- **No Inversion of B or H**
- **Not sensitive to initial guess of X**

Ill-Conditioning & Noisy Data:

- Consider the system $\mathbf{A}\mathbf{X} = \mathbf{b}$

\mathbf{A} : Ill-Conditioned \mathbf{b} is noisy.

Let $\mathbf{A} = \mathbf{U} \text{diag}\{s_i\} \mathbf{V}^T$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{b} = \mathbf{V} \text{diag}\{s_i^{-1}\} \mathbf{U}^T \mathbf{b}$$

- Regularization of Tikhonov [1977]:
- Modify s_i^{-1} for those which are small, compared to others, by: $\omega(s_i^2) s_i^{-1}$

- Tikhonov Filter $\omega(s) = \frac{s}{(s + \alpha)}$ α :Regularization Parameter

- Hansen & O'Leary (1993):

$$\min \Phi(\mathbf{X}) = \min \left[\|\mathbf{A}\mathbf{X} - \mathbf{b}\|^2 + \alpha \|\mathbf{X}\|^2 \right]$$

$$\left(\mathbf{A}^T \mathbf{A} + \alpha \mathbf{I}_n \right) \mathbf{X} - \mathbf{A}^T \mathbf{b} = \mathbf{0}$$

- AE to ODE (Liu & Atluri 2008)

$$\mathbf{A}^T \mathbf{A} \mathbf{X} - \mathbf{A}^T \mathbf{b} = 0$$

$$\mathbf{X}' = \frac{d\mathbf{X}}{d\tau} = -\frac{\nu}{q(\tau)} [\mathbf{A}^T \mathbf{A} \mathbf{X} - \mathbf{A}^T \mathbf{b}]$$

Let $t = \int_0^\tau \frac{\nu}{q(\xi)} d\xi$

$$\dot{\mathbf{X}} = \frac{d\mathbf{X}}{dt} = -\mathbf{A}^T \mathbf{A} \mathbf{X} + \mathbf{A}^T \mathbf{b}$$

$$\mathbf{X}(t) = \mathbf{V} \operatorname{diag} \{ \omega(s_i) s_i^{-1} \} \mathbf{U}^T \mathbf{b}$$

$$\omega(s) = 1 - e^{-ts}$$

Let $q(\tau) = (1 + \tau)^\gamma \quad 0 \leq \gamma < 1$

$$t = \nu \ln(1 + \tau) \quad \gamma = 1$$

$$= \frac{\nu}{1 - \gamma} \left[(1 + \tau)^{1 - \gamma} - 1 \right] \quad 0 \leq \gamma < 1$$

$$\omega(s) = 1 - \frac{1}{(1 + \tau)^{\nu s}} \quad \gamma = 1$$

$$= 1 - \exp\left(\frac{-s}{1 - \gamma} \left[(1 + \tau)^{1 - \gamma} - 1 \right] \right) \quad 0 \leq \gamma < 1$$

$$\dot{\mathbf{X}} = \frac{d\mathbf{X}}{dt} = -\left[\mathbf{A}^T \mathbf{A} + \alpha \mathbf{I}_n \mathbf{X}\right] + \mathbf{A}^T \mathbf{b}$$

$$\omega(s) = \frac{1 - e^{-t(s+\alpha)}}{(s + \alpha)} s$$

Filter in the current FTIM

Solved by Group Preserving Scheme

- Ill-posed Linear Problems, Noisy Data & Filtering.

- Differentiation of Noisy-Data

a Crack-length.

$\frac{da}{dN}$: rate of Crack-growth , per fatigue cycle, N

$$\left(\frac{da}{dN} \right) = C(a)^n = d(\Delta K)^n$$

How do you find $\left(\frac{da}{dN} \right)$ from the

a vs N data, which may be noisy?

- Fatigue Crack-Growth: $\left(\frac{da}{dN} \right)$

- We start with experimentally determined a vs N curve with discrete data:

$$a(N_i) = a_i$$

- Consider a Noise

$$\hat{a}(N_i) = a(N_i) + \sigma R(N_i)$$

$$R(N_i) = \text{Random}$$

- Consider the Laplace transform of $\left(\frac{da}{dN} \right)$

- Finding da/dN from Noisy Data for a versus N

a and (da/dN) are assumed to be defined in the interval $[0, N]$

$$\int_0^N e^{-sN} \frac{da}{dN} dN = s \int_0^N e^{-sN} a(N) dN - a(0)$$

$$\left(\frac{\Delta N}{2}\right) K_{i,1} \left[\frac{da}{dN} (N_1) \right] + \Delta N \sum_{j=2}^m \left(\frac{da}{dN} \right)_{ij} + \left(\frac{\Delta N}{2}\right) K_{i,m+1} \left[\frac{da}{dN} \right] (N_{m+1}) = h[N_i]$$

$$N_i = (i-1)\Delta N = (i-1)\frac{N}{m} \quad s_i = (i-1)\Delta s$$

$$K_{ij} = \exp[-s_i N_j]$$

$$\begin{aligned} h(N_i) = & s_i \left[\frac{\Delta N}{2} K_{i,1} \hat{a}(N_i) \right. \\ & + \Delta N \sum_{j=2}^m K_{i,j} \hat{a}(N_j) \\ & \left. + \frac{\Delta N}{2} K_{i,m+1} \hat{a}(N_{m+1}) - \hat{a}(N_i) \right] \end{aligned}$$

$$\begin{aligned}
[A] \left\{ \frac{da(N_i)}{dN} \right\} &= h(N_i) \\
&= s_i \left[\frac{\Delta N}{2} K_{i,1} \hat{a}(N_i) \right. \\
&\quad + \Delta N \sum_{j=2}^m K_{i,j} \hat{a}(N_j) \\
&\quad \left. + \frac{\Delta N}{2} K_{i,m+1} \hat{a}(N_{m+1}) - \hat{a}(N_1) \right]
\end{aligned}$$

$$\hat{a}(N_i) = \text{given} = a(N_i) + \sigma R(N_i)$$

$$\mathbf{AX} = \mathbf{b} \quad \mathbf{X}' = \frac{d\mathbf{X}}{d\tau} = -\frac{\nu}{1+\tau} \left[\mathbf{A}^T \mathbf{AX} - \mathbf{A}^T \mathbf{b} \right]$$

Solved by Group Preserving Scheme

- Example:

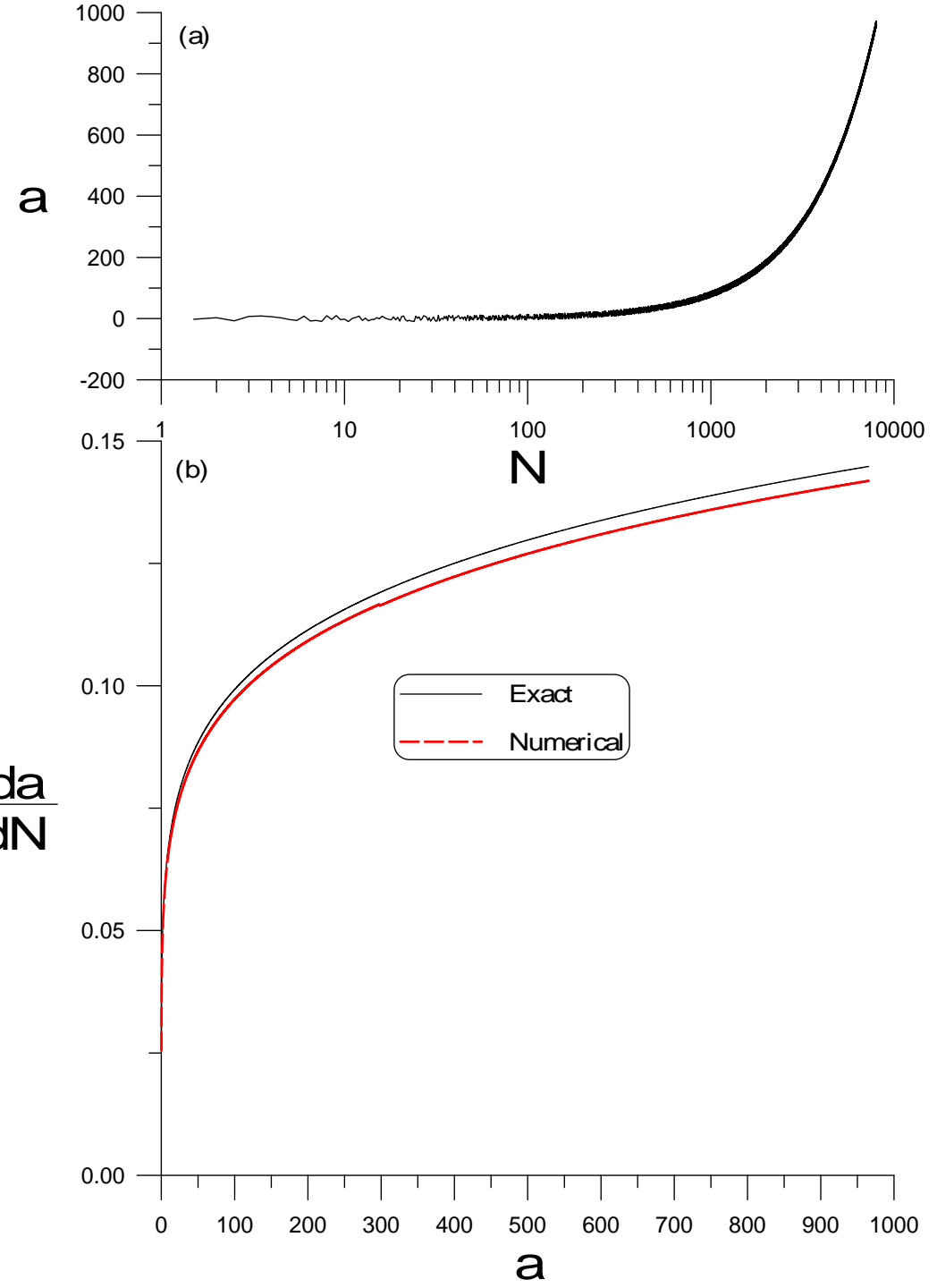
$$\Delta\tau = 10^{-4}$$

$$m = 1000$$

$$\nu = 0.05$$

$$r = 1$$

Comparison of the numerical and exact solutions for obtaining da/dN versus a from the measured noisy data for crack-length a versus the number of cycles, N .



- Inverse Laplace Transform

$$\int_0^{\infty} e^{-st} f(t) dt = F(s)$$

Given $F(s)$, find $f(t)$

Let $f(t) = \sum_{j=1}^n C_j \left(\frac{t}{T_0} \right)^{j-1}$

$$C_1 + \sum_{j=2}^n \frac{(j-1)! C_j}{(T_0 s)^{j-1}} = sF(s)$$